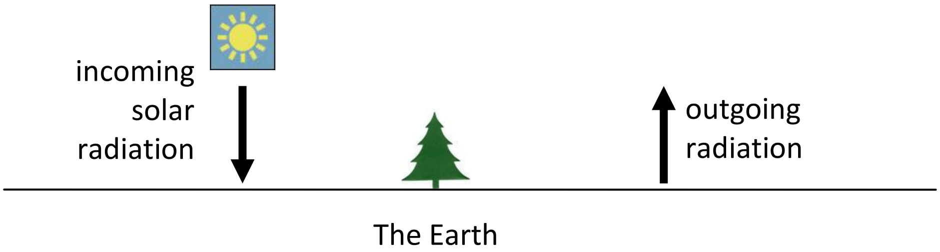
In this activity you will use your graphic calculator to model what would happen to the temperature of the Earth if there were to be a sudden change in the amount of radiation entering or leaving the planet. You will then investigate which functions best model the results you obtain.

Information sheet

At the most fundamental level, the temperature of the Earth is governed by the difference between the amount of **energy** the Earth receives from the Sun, and the amount of energy the Earth loses to Space.



The **outgoing radiation** depends only on the temperature of the Earth, T kelvins (K).

The amount of outgoing radiation is given by the Stefan-Boltzman Law:

Outgoing radiation = $\boldsymbol{\sigma T^4}$ where **$\boldsymbol{\sigma}$ = 5.67 × $\mathbf{10^{-8}}$** $\text{Js}^{-1}\text{m}^{-2}\text{K}^{-4}$

Assume that the (average) temperature of the Earth's surface is 283 K.

Then the outgoing radiation = σT^4 = 364 $\text{Js}^{-1}\text{m}^{-2}$ (to 3sf).

This is the energy lost per square metre of the Earth's surface per second.

The **temperature** of the Earth depends on the difference between the incoming and outgoing radiation and the heat capacity of the Earth.

If the incoming radiation is equal to the outgoing radiation (in this case 364 $\text{Js}^{-1}\text{m}^{-2}$) then the temperature of the Earth is constant. If the incoming radiation and outgoing radiation are different, then the temperature of the Earth will change.

The following formulae can be applied over successive time increments to predict what will happen to the temperature as time passes.

Change in temperature of the Earth

$$= \frac{(\text{incoming radiation - outgoing radiation}) \times \text{time in seconds}}{\text{heat capacity}}$$

where the heat capacity of the Earth = $\mathbf{4 \times 10^8}$ $\text{JK}^{-1}\text{m}^{-2}$

New temperature of the Earth = old temperature + temperature change

Over its lifetime the Sun's luminosity, the amount of energy it emits each second, has increased and it will continue to do so. Suppose that the Sun suddenly increased its luminosity by 5% and then stayed the same.
What would happen to the Earth's temperature? How long would this take?

Think about…

- Check you can do this calculation: the outgoing radiation = σT^4 = 364 $\text{Js}^{-1}\text{m}^{-2}$ (to 3sf)
- What is the kelvin temperature in Celsius?
- How do you enter numbers in standard form into your calculator?
- In general terms, how would you expect the Earth's temperature to change after a sudden increase in radiation input?

Try these ...

In these activities you will use a graphic calculator to predict what will happen if the incoming radiation changes from the equilibrium value of 364 $\text{Js}^{-1}\text{m}^{-2}$ when the temperature of the Earth is 283 K. Use the information and formulae given above.

Investigate an increase in incoming radiation

1 What do you think could change the amount of incoming radiation?

2 Consider the case where the incoming energy increases by 5% and then remains constant.

Now incoming energy = 1.05 x 364 = $\boxed{\qquad\qquad \text{Js}^{-1}\text{m}^{-2}}$

3 Use the formulae given above to complete the table on the next page for this new situation.

Time x (years)	Incoming radiation ($Js^{-1}m^{-2}$)	Outgoing radiation ($Js^{-1}m^{-2}$)	Change in temperature (K)	Temperature of Earth y (K)
0	364	364	–	283
1	382.2			
2	382.2			
3	382.2			
4	382.2			
5	382.2			
6	382.2			
7	382.2			
8	382.2			

4 Plot a graph of temperature against time on your graphic calculator.

5 Describe what happens to the temperature of the Earth.

6 Find one or more polynomial functions to model the temperature data.

To do this, select a quadratic (X^2), cubic (X^3) or quartic (X^4) regression line on your graphic calculator.

7 Compare your model(s) with an exponential model: $y = 283 + 3.53(1 - e^{-0.54x})$.

8 Evaluate the functions as models of the temperature.
Consider each of the functions you have found (quadratic, cubic and quartic) and the exponential function as models of this situation.
For each model answer the following questions.

a How well does the model fit the temperature data?

b Do you think that the model gives realistic predictions for later times?

Investigate a decrease in incoming radiation

Consider the case where the incoming energy decreases by 5% and then remains constant.

Now incoming energy = 0.95 x 364 =

9 Complete the table below for this new situation.

Time (years)	**Incoming radiation ($Js^{-1}m^{-2}$)**	**Outgoing radiation ($Js^{-1}m^{-2}$)**	**Change in temperature (K)**	**Temperature of Earth (K)**
0	364	364	–	283
1				
2				
3				
4				
5				
6				
7				
8				

10 Plot a graph of temperature against time on your graphic calculator.

11 Describe what happens to the temperature of the Earth in this case.

12 Find one or more polynomial functions to model the temperature data.

To do this, select a quadratic (X^2), cubic (X^3) or quartic (X^4) regression line on your graphic calculator.

13 Compare your model(s) with an exponential model: $y = 279.45 + 3.55e^{-0.5x}$.

14 Evaluate the functions as models of the temperature.
Consider each of the functions you have found (quadratic, cubic and quartic) and the exponential function as models of this situation. For each model answer the following questions.

a How well does the model fit the temperature data?

b Do you think that the model gives realistic predictions for later times?

Extension

If you have time, find models for other percentage increases and decreases.

You could also try using different time increments.

You could investigate an ongoing small percentage increase in radiation.

Reflect on your work

Why does an exponential function give a better long-term prediction than any of the polynomials?